Fixed Points of DDS

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06 Maret 2018

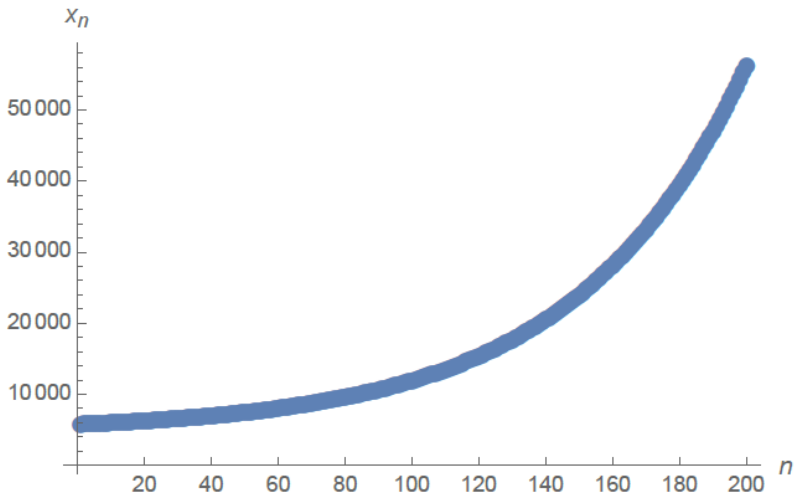
Key Words and Concepts

- Fixed point
- Attracting / Repelling / Neutral
- Fixed point diagram
- Periodic point

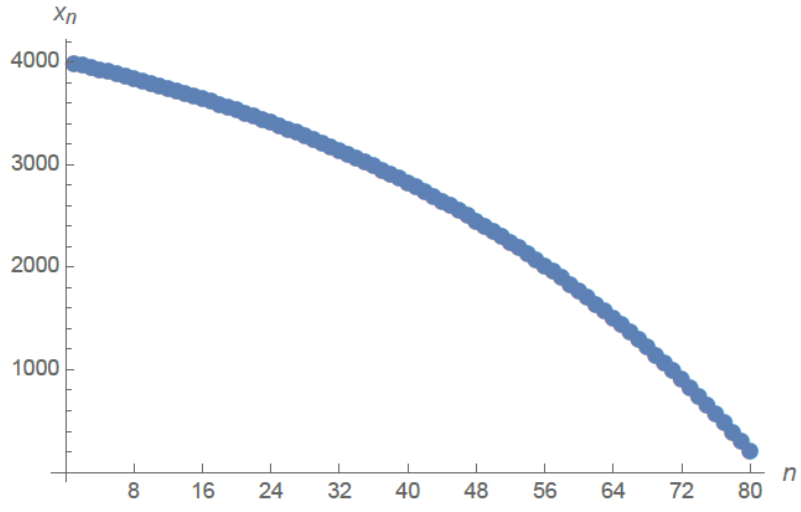
Questions to Consider

- Why do we care about fixed points?
- How can we find fixed points?
- What does the ”stability” of a fixed point mean? How can we determine a fixed point’s stability graphically and symbolically? What are the pitfalls of each method?
- How to answers to the questions above change when considering periodic points instead of fixed points?

Fish Population with Harvesting

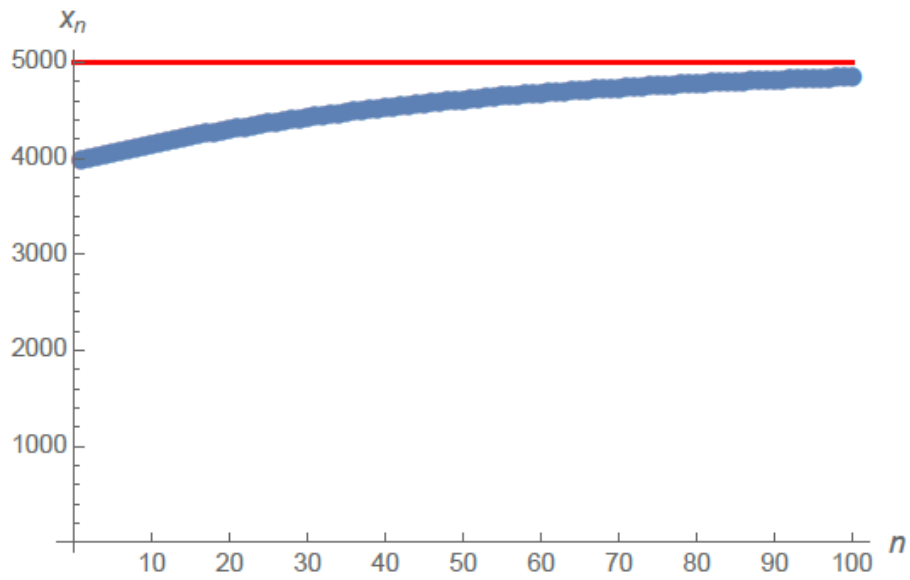


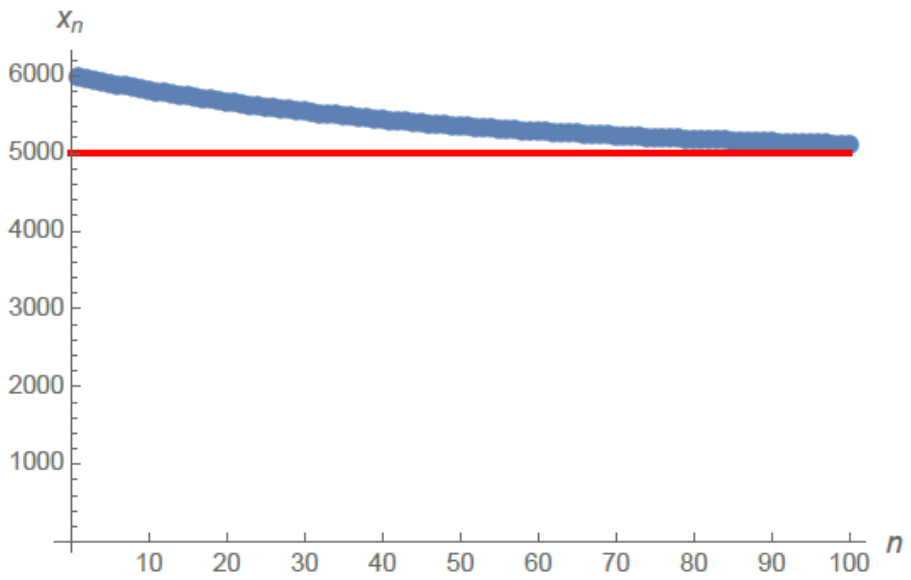
Initial Seed 6000



Initial Seed 4000

Fish Population with Restocking





Stability of Fixed Points

Given a DDS $x_{n+1} = F(x_n)$, we say the fixed point x^* is

- **Attracting** if there exists an interval centered at x^* so that every seed x_0 in the interval has its orbit stay inside the interval and $x_n \to x^*$.

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- **Neutral** if the point fails to qualify as attracting or repelling.

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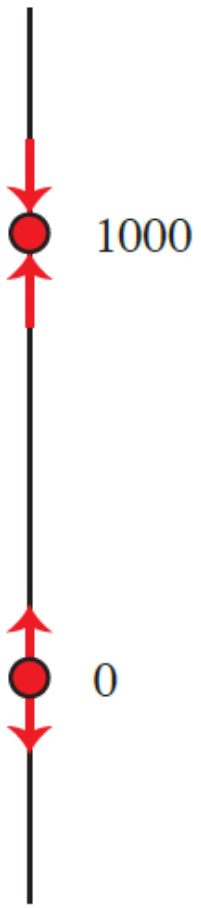

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5. $x_{n+1} = 0.99x_n + 10x_n^2 \sin(1/x)$

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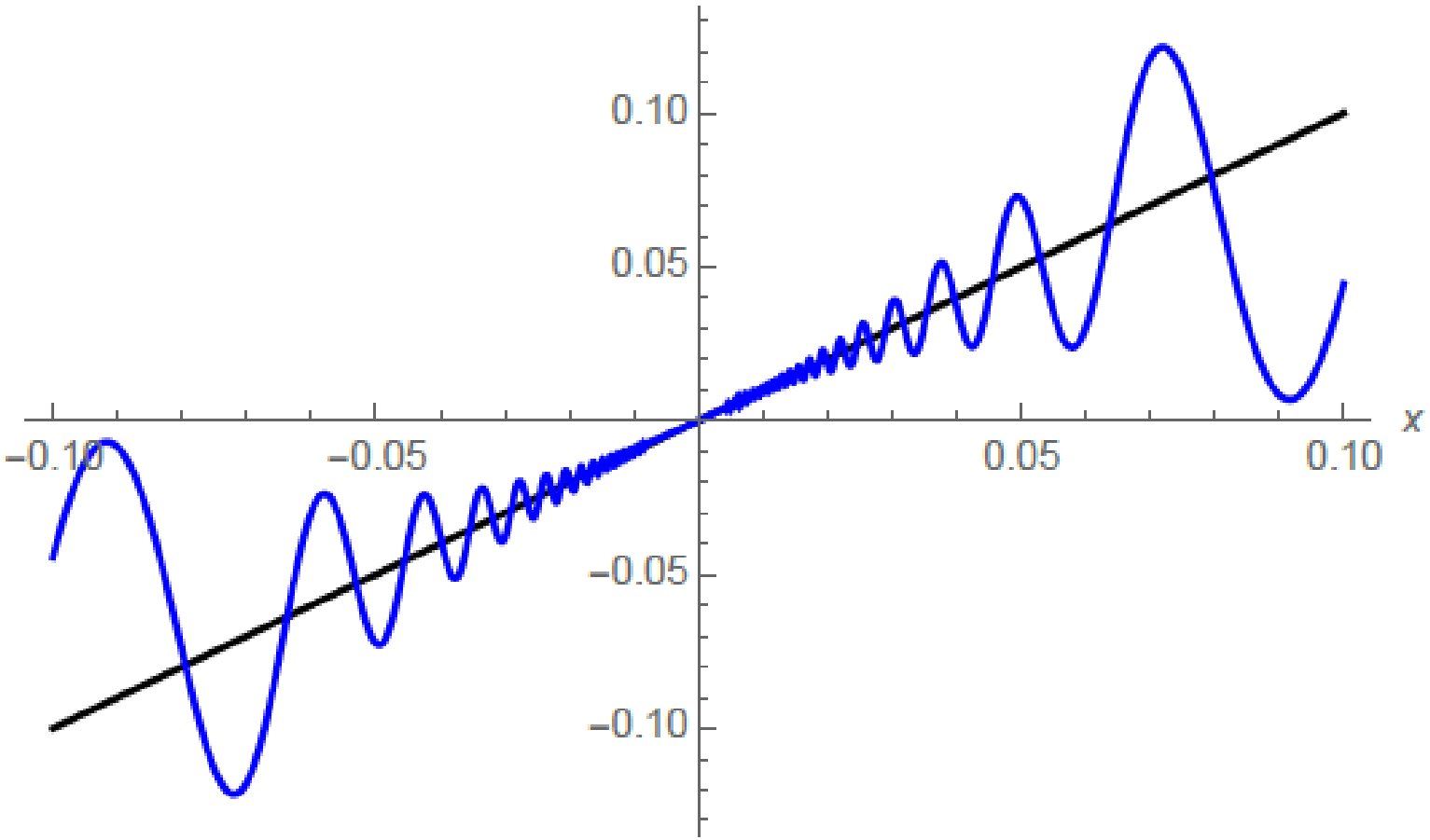

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- If $\left|\frac{dF}{dx}(x^*)\right| > 1$, then x^* is repelling
- If $\left|\frac{dF}{dx}(x^*)\right| = 1$, then x^* could be attracting, repelling, or neutral.